Household Demand System Analysis: Implications of Unit Root Econometrics for Modeling, Testing and Policy Analysis

Working with time-dependent information (series that contain unit roots) has strong implications for consumer analysis. A change in consumer policy that is intended to have a transitory impact on household decisions may have a permanent impact, and estimated parameters of decision making models may not follow standard distributions.

Jonathan J. Fox, The Ohio State University[1]

Introduction

Time series data has been heavily relied upon for inference throughout the history of consumer analysis (Stone, 1954; Barten, 1977; Deaton and Muellbauer 1980). However, time series such as household expenditures and prices, have not been shown to be time homogeneous. If the distribution of variables such as expenditures on food and clothing are time-dependent (contain unit roots), then their mean and/or variances are functions of time. Time dependency, or nonstationarity, has remarkable implications for both consumer policy and statistical analysis. Firstly, in the presence of unit roots, any transitory policy change may have a permanent effect on the variable of interest. Secondly, in the presence of unit roots in the data, estimated parameters of demand systems would not follow the usual standard (textbook) distributions; hence, previous empirical findings would be suspect.

This paper provides an expository discussion of the research methodology used in Fox (1994). The analytical approach described in this paper contrasts with those from previous empirical studies by focusing on the time series properties of the data. It is argued that analytical techniques, which focus on these time series properties, provide the best link between consumer theory and short-run information (data) on consumer activity. For the most part, previous studies of consumer demand have ignored the time series properties of the data, therefore, these models of demand can be improved. Unit root testing, which leads to tests for cointegration (long-run equilibrium), has been shown to be an efficient way to make use of the full amount of information in each time series (Mokhtari, 1992; Banerjee, Dolado, Galbraith and Hendry, 1993).

Time Series Reflecting Consumer Behavior

Current and constant (1982) dollar personal consumption expenditures, ranging from 1946 to 1990, for eleven expenditure categories were obtained from the U.S. Department of Commerce, Bureau of Economic Analysis entry in The Economic Report of the President (1991). From these series, the price series used by the U.S. Department of Commerce are easily obtained by dividing current expenditures by constant expenditures for each year for each category. The price series, used in conjunction with current expenditure series and a total expenditure series, provide the essential variables for a theoretically plausible model of consumer demand.

A Graphical Analysis

A great deal can be learned about consumer behavior by looking at the time paths for both the levels and the first differences of key consumption variables. In most cases, the graphs of the levels for budget shares and prices show a distinct trend, along with cyclical variation. The ups and downs of gas and motor vehicle prices are obvious to both the individual consumer and the economist tracking these series. What is not so obvious to the individual, but blatantly obvious to researchers who graph their data, are the stochastic trends imbedded in most of the series used in the analysis of demand.

The time paths for food provide a typical example of a downward trending series (see Figures 1 and 2). The graph of the one period change in food expenditures shows that this downward trend is not simply a constant or a deterministic trend. The jagged paths traveled by the one period changes series indicate that the downward trend follows a variable or stochastic process. For example, Figure 2 shows the rate of decline in the portion of the budget spent on food tends to hover around a half of a percent, but, the actual rate of decline is rarely -.005, as would be implied by a deterministic trend. In fact, for every series, the changes in the variable appear to exhibit a degree of bounded variability. Thus, upon a simple graphical representation of the data, it becomes clear that this stochastic movement of the changes in each series needs to be included in the model of consumer demand.

All of the price variables, measured on a logarithmic scale, exhibit an upward trend over the period of analysis. Again, evidence from the time paths of the changes in prices shows that the upward trend is not a constant. There appears to be a stochastic element in the upward trend. The time paths of the one period changes in the natural log of prices, which is the growth rate in prices, are generally very different. For example, the growth rate of the price of clothing moves about erratically in the early 1950s, tends to increase over the 1960s and 1970s, and stabilizes in the 1980s while the growth rate of the price of food tends to be more erratic in the 1970s and early 1980s.

Many of the interesting features of the time paths of the variables commonly used to describe the consumer decision making process coincide with a specific event or period in history. The fact that the impact of historical events can be seen in the data may be the single most appealing aspect of using time series data to analyze consumption. For example, the beginning of

Figure 1
Food Budget Shares

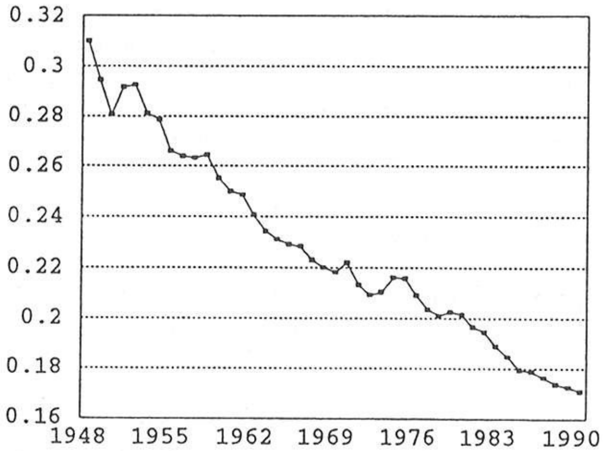
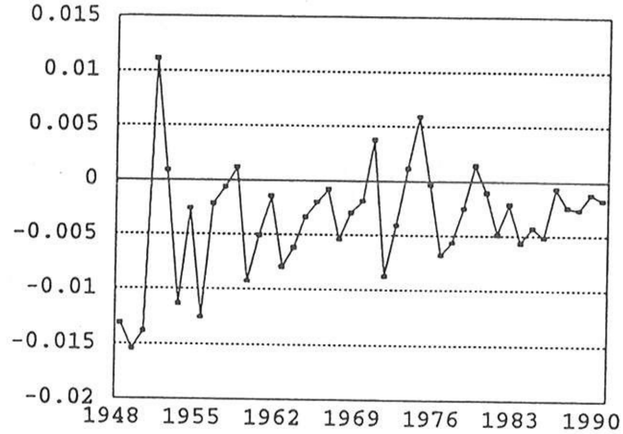


Figure 2
Change in Food Budget Shares



Medicare in 1966 precedes a period of erratic medical expenditures while the 1973 Arab oil embargo appears to have significantly influenced several of the series. It is well worth stressing that the clear trends or apparent nonstationarity of the levels of most of these variables carries the impact of these and many other historical events (shocks) as a permanent component of their time series behavior. Furthermore, it is also the case that this historical information is all but lost when only the first differences of the series are used in analysis.

Formal Tests for Unit Roots

Simply looking at the time paths of each series is an excellent first indication of the time dependency, or nonstationarity, of a process. However, a formal test of nonstationarity is desirable. In practice, there are several formal tests for unit roots (nonstationarity). Among the most popular of these tests are the Cointegration Regression Durbin-Watson test of Sargan and Bhargava (1983), the Dickey-Fuller (DF) test, and the augmented Dickey-Fuller (ADF) test. The ADF is the most widely used test for a unit root, due to its simplicity and efficiency.

To understand the Dickey-Fuller approach to testing for unit roots it is helpful to consider the autoregressive model,

$$X_t = \beta X_{t-1} + \epsilon_t, \qquad (1)$$

where t=1,2,...T. Equation (1) describes a process that began long ago, where β is a constant and ϵ_t is a normally distributed, random variable with a mean of zero and a variance of σ^2. The series X_t is stationary if $|\beta|<1$. If $|\beta|=1$, then the series is not stationary, and in the case where $|\beta|>1$, the process X_t is said to be explosive. Most economic series have been found not to demonstrate an "explosive" tendency (Nelson and Plosser, 1982), and, thus, it is common practice to test the null of $\beta=1$ against the one-tailed alternative of $|\beta|<1$. In practice, the null of $\beta=1$ is often incorporated through the first difference of a series,

$$X_t - X_{t-1} = \epsilon_t, \quad (2)$$

which is merely a rearrangement of equation (1). Otherwise, X_{t-1} can be reduced from both sides of equation (1) to obtain

$$X_t - X_{t-1} = \beta X_{t-1} - X_{t-1} + \epsilon_t; \quad (3)$$

thus,

$$\Delta X_t = (\beta-1)X_{t-1} + \epsilon_t \quad (4)$$

or

$$\Delta X_t = \phi X_{t-1} + \epsilon_t, \quad (5)$$

where, $\phi = (\beta-1)$. Ordinary least squares estimates of ϕ can be used to test the null of nonstationarity for X_t, $\beta=1$, or a unit root. The ratio of ϕ to its standard error yields the essential test statistic of the Dickey-Fuller test. If $\phi=0$ in equation (5), then equations (5) and (2) are the same, and the null of nonstationarity, or of a unit root, is not rejected. However, testing for the null of $\phi=0$ poses a unique problem. The distribution of the test statistic does not follow a standard distribution when integrated process are included in the regression (Granger & Newbold, 1974; Phillips, 1986; Davidson & MacKinnon, 1993). Dickey and Fuller (1979) show that the OLS estimates for β are distributed around a value which is in fact less than unity ($\phi<0$). The t-ratio for ϕ does not have a limiting normal distribution. The distribution is so negatively skewed that most of its mass falls below zero. Therefore, the critical values in the left-hand tail are less than those that would be observed in the conventional Student-t distribution. Thus, the usual t-tests are not appropriate for testing the null hypothesis of a unit root ($\beta=1$ or $\phi=0$). Dickey and Fuller, along with others have used Monte Carlo simulations to calculate tables of critical values based on the distributions of the statistics gained from such tests for nonstationarity. These resulting tables of critical values are, in absolute value, higher than normal critical values.

The augmented Dickey-Fuller test is a generalization of the simple Dickey-Fuller test. The general representation of the ADF regression is

$$\Delta X_t = \alpha_1 + \alpha_2 t + \phi X_{t-1} + \Sigma \beta_i \Delta X_{t-i} + \epsilon_t, \quad (6)$$

where t is a time trend. Equation (6) should include as many lagged values of the dependent variable (ΔX_{t-i}) as are necessary to ensure that ϵ_t is white-noise. A Lagrange Multiplier test can be used to test the residuals in equation (6) for serial correlation to ensure that enough lagged values of the dependent variable have been included. Accounting for serial correlation provides a major improvement over what would be an invalid DF-test in the presence of serial correlation. The value of the ratio of ϕ over its standard error in equation (6) is, again, the essential element in the test of the null of nonstationarity, $\phi=0$. Since the serial correlation in the error terms has been fully accounted for by the inclusion of lagged values of the dependent variable in equation (6), the same asymptotic critical values calculated for DF-tests can be used to test the null of nonstationarity (Davidson and MacKinnon, 1993, p. 711). For negative values of ϕ, if the calculated value of the of ratio ϕ to its standard error is larger (in absolute value) than the critical value from the table, which are generally greater than 3, then we reject the null of a unit root (nonstationarity).

Implications of Nonstationarity

For Policy Analysis

To highlight the implications of a unit root in a series, it is helpful to consider the case where the series X_t has been found to be integrated of order one. Thus, X_t can be written as

$$X_t = X_{t-1} + \epsilon_t \text{ or } \Delta X_t = \epsilon_t. \quad (7)$$

Similarly, any autoregressive process that has a coefficient ρ equal to one in the equation

$$X_t = \rho X_{t-1} + \epsilon_t \quad (8)$$

is described as having a unit root. Assuming ϵ_t is generated by a white noise process with variance σ^2, then (8) is also known as a simple random walk or Martingale process. Repeated substitution of $X_{t-1} = \rho X_{t-2} + \epsilon_{t-1}$, $X_{t-2} = \rho X_{t-3} + \epsilon_{t-2}$, ... into equation (8) yields

$$X_t = \textstyle\sum_{j=1}^{T} \epsilon_{t-j}. \quad (9)$$

If ϵ_t is considered to be the result of a shock (*e.g.* a policy change) to this process, then it is clear that this disturbance will have a permanent impact on the series. This is the reason that the term "integrated" is used to describe X_t, which amounts to an accumulation of persistent shocks when a unit root exists.

On the other hand, if X_t was generated by a stationary process, *i.e.*, $|\rho|<1$ in equation (8), then similar repeated substitutions produce

$$X_t = \sum_{j=0}^{T} \rho^j \epsilon_{t-j}, \qquad (10)$$

which clearly shows that the effect of the shocks to the system decrease over time. The significance of this "persistent *versus* diminishing" effect of shocks to a system cannot be overemphasized. If policy recommendations are to be made on the basis of such series, then it is of vital importance to find out whether the variable of interest is a stationary or a nonstationary process. If a nonstationary process is subjected to a shock, then the impact of this shock will be permanent. The repercussions of the policy change will remain in the system, at the strength of the initial impact, forever. Thus, the problems facing the consumer policy analyst are further complicated by permanent changes in the underlying process that she is trying to describe or influence. Therefore, a significant degree of emphasis must be placed on tests for unit roots (nonstationarity) in any policy analysis that uses economic time series.

For Statistical Analysis

The statistical implications of analyzing variables that contain unit roots are as profound and significant as those for policy. Although the mean of a nonstationary process, X_t, may be zero, $E(X_t)=0$, its variance is not constant over time. Using the same repeated substitution used to describe a nonstationary process as an accumulation of persistent errors, the variance of a nonstationary process can be expressed as

$$\begin{aligned} Var(X_t) &= Var(\epsilon_t)+Var(\epsilon_{t-1})+...+ \\ &Var(\epsilon_{t-T}) = \sigma^2 T, \end{aligned} \qquad (11)$$

where it is shown to be growing over time. The variance is no longer a function of time when $|\rho|<1$:

$$\begin{aligned} Var(x_t) &= \rho^0 Var(\epsilon_t)+\rho^1 Var(\epsilon_{t-1})+ \\ &...+\sigma^T Var(\epsilon_{t-T}) \\ &= \sigma^2(1 + \rho + \rho^2 + ... + \rho^T) \\ &= \sigma^2/1-\rho^2. \end{aligned} \qquad (12)$$

The implications for statistical inference are profound, as most aspects of statistical inference rely upon the variance of a process. Once the variance of a process is determined to be a function of time, it can no longer be used reliably in statistical inference.

With respect to modeling with integrated processes, Granger and Newbold (1974) show that test statistics frequently confirm a relationship between integrated variables, even when the processes considered could not possibly be related. Granger and Newbold examine time series which are known to be generated by random walks. They find that when they estimate these spurious regressions between integrated processes, the value of R^2 turns out to be high, and the value of the Durbin-Watson statistic tends to be low, regardless of the existence of any true relationship between the variables. Furthermore, the t-statistics for the estimated regression coefficients imply a rejection of the null of no relationship far more often than is expected.

Phillips (1986), expounding upon Granger and Newbold, considers the basic regression model

$$Y_t = \alpha + \beta X_t + u_t, \qquad (13)$$

where $u_t \sim (0,\sigma_u^2)$ and Y_t and X_t follow a random walk. Therefore,

$$Y_t = Y_{t-1} + \epsilon_t \qquad (14)$$

and

$$X_t = X_{t-1} + \upsilon_t. \qquad (15)$$

Phillips shows that as the number of observations approaches infinity, the t-statistics for β and the value of R^2 in equation (13) approach random variables, while the Durbin-Watson statistic approaches zero. Davidson and MacKinnon (1993) carry out Monte Carlo simulations to illustrate the increasing number of rejections of the null of $\beta=0$ in equation (13) as the sample size increases. With a sample size of 25, the null is rejected 53% of the time. With a sample of 2000, the null of $\beta=0$ in equation (13) is rejected 94.7% of the time. Furthermore, if a drift or a trend is added to the model, the null is rejected 100% of the time after attaining samples of 500 and 100 respectively (Davidson and MacKinnon, 1993, p.672).

Davidson and MacKinnon further point out that rejecting the null of $\beta=0$ does not imply that $\beta\neq0$ in (13); it only implies that the null is false. If Y_t is a random walk process, it certainly cannot equal, as (13) implies if $\beta=0$, a constant plus a stationary error term; therefore, the null is false to begin with, and the test is invalid from the start. The only way that the equality expressed in (13) can be satisfied when $\beta=0$ is if u_t follows a random walk. Furthermore, if u_t is a random walk (which it is assumed not to be), then the t-value becomes a function of this integrated process. For that matter, any and all statistics that are functions of these disturbances become functions of random walks (or more precisely, functions of Wiener processes). Obviously, reliable inference cannot be based on statistics that are functions of integrated processes, as these processes are known to have a variance which is growing over time. Given that the model in equation (13) is misspecified, Davidson and MacKinnon estimate

$$Y_t = \alpha_1 + \beta X_t + \alpha_2 Y_{t-1} + u_t, \tag{16}$$

where (14) and (15) still apply. They find that the null of $\beta=0$ is rejected about 15% of the time, regardless of sample size (Davidson and MacKinnon, 1993, p.672). This result is drastically different from that obtained when the lagged value of the dependent variable is not included in the regression. However, 15% is still three times greater than the usual and acceptable 5% rate of committing a type-one error. This discrepancy further demonstrates the way in which t-statistics from regressions including integrated processes follow non-standard distributions asymptotically. It is clear that a researcher must obtain distributions of the estimates through alternative means (*e.g.* Monte Carlo experiments), in order to conduct reliable inference when integrated processes are involved.

For Modeling

Ultimately, the statistical problems associated with using nonstationary series can be circumvented through the use of a dynamic modeling strategy which relies on tests for cointegration and the estimation of error correction models (ECM). The ECM framework uses only stationary series or linear combinations of series which are stationary; moreover, the ECM model is dynamic in nature and uses both the long-run and the short-run information embedded in trended variables, such as expenditures and prices.

To illustrate the concept of cointegration, which is the cornerstone of the ECM framework, it is helpful to consider a simple equilibrium relationship which can be defined as

$$Y_t = \beta X_t. \tag{17}$$

If Y_t follows an equilibrium path with X_t, then

$$Y_t - \beta X_t = 0. \tag{18}$$

However, it is not expected that equation (18) will be satisfied at all times, even if Y_t and X_t are closely related in the long-run. In fact, most of the time, this equality will not be satisfied. This is the case with most economic processes. For example, people rarely spend exactly what they earn; in fact, people rarely spend what they expect or desire to spend. Thus, to illustrate the frequent periods of disequilibrium which clearly take place, it is helpful to write equation (18) as

$$Y_t - \beta X_t = \epsilon_t, \tag{19}$$

where ϵ_t is the disequilibrium error. Nonetheless, equation (19) shows that the parameters $[1,-\beta]$ work to maintain a constant and unique relationship between Y_t and X_t. Engle and Granger (1987) maintain that if an equilibrium relationship exists, then the values of ϵ_t, or the disequilibrium error, should not be large; in fact, the disequilibrium error should demonstrate a strong tendency to become very small as time passes. Thus, the variables must not drift too far apart when scaled. In this case, the scaled transformations placed on Y_t and X_t are 1 and $-\beta$, respectively. This transformation is called the scaled cointegrating vector $[1,-\beta]$. Accepting that equilibrium relationships are essentially two or more series which are cointegrated, Granger (1983) advocates the use of an error correction mechanism to illustrate the short-run adjustment process toward equilibrium. Thus, Granger argues that an error correction mechanism should be used when modeling in the presence of cointegration, an argument that has become known as the Granger Representation Theorem.

The first requirement in testing for cointegration is that the series in question, must share some level of integration. For example, a series that is drastically trending downward cannot be meaningfully explained by a series that hovers around a constant. Therefore, in the simple regression model

$$Y_t = \beta X_t + \epsilon_t \tag{20}$$

where $\epsilon_t \sim IIN(0,\sigma^2)$,there will only be a parameter β that satisfies equation (20) if Y_t and X_t are integrated of the same order (*i.e.* requiring to be differenced the same number of times to be made stationary).

The second requirement for cointegration between two series implies that $(Y_t-\beta X_t)$ is integrated to a lower order than Y_t and X_t. This can be tested by testing for the nonstationarity of ϵ_t or changes in ϵ_t in equation (20). Even though Y_t and X_t may exhibit trends, cyclical or seasonal variation, these changes may be closely matched between the series so that, when scaled by $[1,-\beta]$, the difference between the variables shows no such trend or cycle.

If Y_t and X_t are integrated of order one, and (Y_t-βX_t) is stationary, then the error correction representation is the proper statistical representation for the relationship, and it contains information on the long-run equilibrium path as well as the short-run adjustment process. A simple error correction representation is

$$\Delta Y_t=\delta+\beta_0\Delta X_t+\lambda(Y-\beta X)_{t-1}+u_t, \qquad (21)$$

where the significance of λ, known as the loading factor, can be used in a further test of cointegration between Y_t and X_t. The loading factor, λ, may capture the speed of adjustment with which consumers are reacting to disequilibrium error. If λ is not significantly different than zero, then equation (21) will contain only short-run information. It is worth noting that equation (21) is easily derived from the simple autoregressive distributed lag model,

$$Y_t=\delta+\alpha Y_{t-1}+\beta_0 X_t+\beta_1 X_{t-1}+u_t, \qquad (22)$$

where $|\alpha|<1$, $\lambda=\alpha-1$, and $\beta=(\beta_0+\beta_1)/(1-\alpha)$ which is the long-run response. If, upon estimation, it is found that the value of α is close to one, and that, therefore, the value of λ is close to zero, then it would be appropriate to estimate the model in first differences only. However, if α is found to be less than one, then a model made up only of first differences will be inadequate with respect to both estimation and forecasting (Harvey, 1990, p.291). In this regard, the term in levels in (21), the error correction term, becomes critical in the description of how the relationship returns to its steady-state or equilibrium. For example, if, at some point, Y_t is subjected to some shock or set of positive disturbances, which cause the series to grow more rapidly than is expected, then the error correction term, $(Y-\beta X)_{t-1}$, becomes positive. If λ is negative, then the net effect is a decrease in the growth of Y_t. Therefore, Y_t is being forced back to its long-run path (Harvey, 1990, p.292).

It is important to emphasize that equations (20) and (21) yield two distinct types of information. Equation (20) gives estimates for the parameters of the long-run equilibrium relationship that is theorized to exist. On the other hand, the estimation of equation (21) yields information on the size and direction of the dynamic adjustment process, through the estimation of β_0 and λ. However, because (20) is imbedded in (21), the estimation of (21) will provide information on both the long-run and the short-run behavior of the variables. This is perhaps the single greatest advantage to using the cointegration framework for modeling consumer behavior. The theory of consumer behavior lacks a time dimension, while the data used in testing this theory is generated over, and is frequently a function of, time.

Conclusion

This paper suggests that the use of unit root econometric techniques may be the best solution to the paradox of using short-run, or dynamic, information to analyze theories that describe long-run, or steady state, relationships. Unit root econometric techniques entail a less conventional modeling approach which involves a search for equilibrium relationships that may exist between certain groups of variables, even though short-run observations on these variables may be time heterogeneous.

It has also been shown that in the presence of unit roots, any transitory policy change may have a permanent effect on the variable of interest. In the case of the consumer, this may imply that information is not forgotten once embedded in the decision making process. Thus, policy makers and consumer educators may need to be even more cautious when implementing programs.

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Endnotes

1. Assistant Professor, Consumer and Textile Sciences