The Effect of Household Size Changes on Credit Use: An Expected Utility Approach

The paper uses a two period expected utility model with uncertainty to provide hypotheses related to empirical analyses of the effect of household size on credit use. The effects of different household size changes, levels of risk aversion and uncertainty in income growth on optimal credit use are demonstrated. The simple analyses presented in this paper can give insights into appropriate empirical tests, and also some implications for optimal credit and saving behavior.

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Introduction

Since the pioneering article by Modigliani and Brumberg (1954), the life cycle model has been the fundamental analysis for consumption and saving behaviors (Wang, Mok, & Hanna, 1994). One of the essential points of the life cycle approach is the recognition that a consumer has a time preference regarding consumption (Magrabi, Chung, Cha & Yang, 1991). The household is presumed to alter the amount and kinds of assets and debts it possesses in the face of changes in the economic environment and changes within the household (Bryant, 1990). One of the biggest changes that can occur within a household is change in household size and household composition. Thus, household size is one of the major factors influencing the time preference or the personal discount rate of the future. Godwin's (1998) empirical study found that household size is a significant factor affecting indebtedness.

The purpose of this paper is to analyze the effect that planned household size change has on optimal saving or borrowing behavior. Households at different life cycle stages should be motivated to save or dissave according to their practical needs and long-term financial plans. Change in household size might come unexpectedly but in many cases, it is the result of anticipated events. Thus, understanding the effect of change in the household size on the optimal amount of saving/borrowing would not only enhance the understanding and explanation of the saving/borrowing behavior but also could help families to manage better by providing some guideline for planning.

Literature

Fisher (1956) was the one of the first economists to call attention to the absence of families in the life cycle model by saying that the theory might be described as “the bachelor theory of Saving”. In reality, the household size usually does not remain constant over its lifetime. Generally, household size increases due to marriage and growing number of children, then remains constant for a while until the children are old enough to leave the home, and then household size diminishes. White (1978) suggested that in order to incorporate changes in family size in calculating the optimal lifetime consumption, equivalence scales should be used in the utility function of the head of the household.

Equivalence scales have been used to enable comparisons of welfare or real income across households of different sizes and composition (Deaton & Muellbauer, 1980). One of the most commonly used equivalence scales for adjusting income in order to compare well-being among different sized households is the one implicit in the U.S. poverty threshold (Blaylock & Blissard, 1990). Thus, although imperfect, incorporating the notion of poverty threshold into the analysis to adjust for the change in household size could be reasonable.

White (1978) used the life cycle model to simulate values of aggregate saving for three different time periods and included the effect of household size in a lifecycle model by using consumption equivalence per person for different periods and number of people in the household in constraint and utility function. The result of simulation shows that the inclusion of household size and upward sloping lifetime income streams reduced the simulated values of aggregate savings. DeVaney and Hanna’s (1991) theoretical analysis of the impact of children on savings using a life cycle model simulation suggests that while children are at home, saving should be deferred
because of the higher weight placed on utility from consumption in years with higher household size. A simulation analysis had 1% saving for a 30 year old couple with two children, compared to 4% saving for an otherwise similar childless couple.

Douthitt and Fedyk (1989)'s empirical analysis using the 1982 Canadian Survey of Family Expenditure looked at the expenditure allocation and suggested that families pay for part of the cost of children by reducing savings. Godwin (1998) used Bryant (1990)'s two-time period utility maximization model as the theoretical basis to empirically investigate the changes in household debt quintile using the 1983-1989 Survey of Consumer Finances panel data set. The result of logistic regression, where the dependent variable was the log odds of change in debt, showed that households with more members were significantly more likely to have experienced an increase in the relative debt position, holding income change, time preference, risk aversion, expectation for interest rate and other variables constant.

Other variables affecting optimal saving and borrowing behavior include the expected growth rate of real income (Fan, Chang, & Hanna, 1993), the variance of future income (Chang & Hanna, 1994), the utility function, the real interest rate (White, 1978; Shapiro, 1984) and the discount rate (Skinner, 1985). Thus, in the simulation analysis, certain assumptions were made about these factors.

**A Two-Period Model of Consumption**

The two-period model of consumption with uncertain income growth used by Fan, Chang & Hanna (1992;1993) was used in this paper. This model assumes that a household attempts to maximize the expected value of utility from consumption for two periods. The decision to borrow or to save is made in conjunction with the known first period income. The second period consumption will be a random variable, dependent on the actual value of second period income, which is assumed to be affected by income growth rate, the probability that income growth occurs and real interest rate. It is assumed that there are two states of the world in the second period, and real income either increases or stays constant. The focus of this paper is on credit use, therefore income increases are included in the analysis. Finally, consumers are assumed to repay any loan in full in the second period.

Thus, mathematically the objective function to maximize is shown by Equation (1) below.

\[
\text{Maximize } T = U(C_1) + [p^*U(C_2) + (1-p^*)U(C_2)]/(1+\phi)
\]  

Where the utility function used is a constant elasticity utility function:

\[
U(C_1) = C_1^{1+\epsilon}/(1+\epsilon) \text{ for } \epsilon \neq -1
\]  

\[
U(C_1) = \ln(C_1) \text{ for } \epsilon = -1
\]

Constraints: 

\[
C_1 = I - S
\]  

\[
C_2 = (1+g)I + (1+r)S
\]  

\[
C_2 = I + (1+r)S
\]

In this paper, for simplicity of exposition, we assume that initial net worth is zero. The variables in the equations include total expected utility for lifetime (T), period 1 income(I), income in period 2 in case of income increase ((1+g)*I), consumption in period 1, consumption in period 2 if income increases, consumption in Period 2 if income does not increase (C1, C2 and C2 respectively), saving in period 1(S), growth rate in real income(g), real interest rate, which will be different depending on whether S<0 or S>0 (r), elasticity of marginal utility with respect to consumption(\epsilon), relative risk aversion(\phi), probability of income increase(p) and personal discount factor(\psi).

With certainty (p=1), Equation (7) shows the optimal saving as a proportion of Period 1 income.

\[
S = \frac{C_1 - (1+\phi)^{-(1/\epsilon)}(1+g)}{(1+r)^{-(1/\epsilon)}(1+\phi)}
\]

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The parameter \( \varepsilon \) reflects intertemporal utility (consumption smoothing), and, with uncertainty about income in period 2, relative risk aversion = \(-\varepsilon\). Most studies of intertemporal consumption have used a constant elasticity utility function (Chang, Hanna, & Fan, 1997; Hurd 1989).

Barsky, Juster, Kimball, and Shapiro (1997) presented an experimental measure based on presenting a set of hypothetical questions to a large national sample of adults aged 51 to 61. Barsky et al. (1997) found that 64.6% had a relative risk aversion level \((-\varepsilon)\) between 3.76 and \(\infty\), 11.6% had a value between 2 and 3.76, 10.9% had a value between 1 and 2, and 12.8% had a value between 0 and 1. Hanna and Gutter (1998) reported the results of a web survey suggesting that most respondents had risk aversion between 2 and 10. Hanna, Fan and Chang (1995) reviewed literature suggesting a range of values of the elasticity of marginal utility (\(\varepsilon\)) with respect to consumption from zero to -15. An introspective scenario suggested levels of 2 to 10. Introspective analyses of relative risk aversion imply levels of 1 to 10 (Hanna & Chen, 1997). In this paper, levels of relative risk aversion \((-\varepsilon)\) of 1 to 8 will be used. The nominal interest rate used for credit was 15.65%, the mean rate for variable rate standard credit cards in May, 1999 (Bankrate.com, 1999). If an inflation rate of 3% is assumed, the real interest rate is 14.3%. The real interest rate used for saving was 2%, which may reflect a typical real, after-tax rate if the pretax real interest rate is 3% (Morgan, 1995). In this paper, the U.S. government poverty thresholds are used for calculating the personal discount factor, \(\varphi\). We assume that if household size is expected to remain constant, \(\varphi = 0\). If household size is expected to change, \(1 + \varphi = \) the ratio of the poverty threshold for the current household size divided by the poverty threshold for the future household size. Therefore, for instance, if household size is expected to change from 4 to 2, \(1 + \varphi = 16660/10634 = 1.57\). In terms of Equation (1), utility from consumption in Period 2 is divided by 1.57, so it is weighted by approximately 0.64. In other words, utility from consumption when the household will consist of only 2 people counts 64% as much as utility from consumption when the household consists of 4 people.

Given the assumption that initial net worth is zero, Equation (7) might have to be calculated twice, once with an interest rate for borrowing, and, if the optimal level of S/I is positive, then again with an interest rate for saving. It is possible that the optimal level of S/I might be zero if S/I is positive using the interest rate for borrowing and negative using the interest rate for saving.

For the examples with Period 2 income certain, Equation (7) was used to find the optimal ratio of saving to Period 1 income. For examples with Period 2 income uncertain, Equation (1) and the constraints in Equations (4), (5), and (6) were used for all possible values of S. The value of S that maximized expected utility according to Equation (1) is reported for each combination of household size change and relative risk aversion.

**Results**

The full paper (available at www.hec.ohio-state.edu/hanna/sh/rhacia.doc) includes graphs and tables based on two scenarios with income certainty: constant income and income certain to increase by 50%. Figure 1 shows the case of uncertainty, with \(p = 50\%\) for a 50\% increase in income. If no change in household size is expected, it is always optimal to borrow and the optimal level of borrowing varies little, from 2.5\% to 5\% of Period 1 income, with maximum borrowing at a level of relative risk aversion of about 2.5. For household size increases, saving is optimal for low levels of relative risk aversion, but low levels of borrowing are optimal for values of relative risk aversion above 3.5 for household size change from 2 to 4 and above 1.5 for household size change from 1 to 2.

The optimal amount of borrowing or saving depends on assumptions about the probability of income increase and the income growth rate. For example, for household size change from 1 to 2 and relative risk aversion of 2, optimal amount of saving as a percent of income is about 9\% for \(p=0\%), and just under 0\% for \(p=50\%\) and \(g=50\%). For income certain to increase 50\%, it is optimal to borrow 12\% of income. For relative risk aversion of 6.0, it is optimal to save 2\% of income if income is expected to remain constant, but it is optimal to borrow if the income is expected to increase, even with uncertainty. The optimal amount of borrowing as a percent of income is about 2\% for the case of uncertainty (\(p=50\%\)) and almost 20\% for the case of certainty. The results show that household size by itself has no impact on optimal saving as a percent of income. If household size will remain constant, optimal saving is the same regardless of whether the household size is 1 or 4. However, the result suggested by previous research that larger household sizes are associated with smaller saving and/or greater debt (Mason, 1975; Douthitt & Fedyck, 1989; Godwin, 1988) might be consistent with the theoretical results shown in this paper, if we add the assumption that all other things equal, a household with more people is more likely to expect a decrease in household size in the future. For any given level of relative risk aversion, as we move from the
2 to 4 change to the 1 to 2 change to no change to the 2 to 1 change to the 4 to 2 change, optimal saving as a percent of Period 1 decreases.

Figure 1
Optimal Saving as a Percent of Period 1 Income, with 50% Probability that Income Increases by 50%, by Relative Risk Aversion (−ε) and Planned Change in Household Size between Period 1 and Period 2.

Implications for Future Empirical Research
In order to ascertain whether households are behaving rationally in regard to credit use, it is necessary to control for expected changes in household size, rather than current household size. Ideally, the actual interest rates faced by each household should be measured. It would also be desirable to include a measure of expected income change and the likelihood that income will increase or decrease (Chang & Hanna, 1994; Chang et al., 1997). A pure measure of the elasticity of marginal utility with respect to consumption and/or relative risk aversion would be desirable. The risk measure in the Survey of Consumer Finance is not a good measure of relative risk aversion (Hanna & Chen, 1997) but the questions used by Barsky et al. (1997) could be used.

Implications for Future Theoretical Research
Clearly, the model used in this paper is simplistic, although the results did provide useful insights. A life cycle model could be used, although there is no realistic way of incorporating uncertainty into a life cycle model. The results in this paper could be combined with the results obtained by DeVany and Hanna (1991), who used a life cycle model with certainty, and assumed the purpose of saving was for life cycle smoothing, for retirement. However, if household contributions to retirement accounts are considered separately from credit use, then the general results from the two theoretical results would be that households should save somewhat less for retirement during each year children are at home, and should save more before children are born and after they leave home. In terms of unsecured credit use, similar results can be obtained from the results in this paper, e.g., there should be no credit card balances before children are born, unless substantial income growth is likely. Additional theoretical issues that should be explored include consideration of alternate equivalence scales, inclusion of more than two states of the world in period 2, and extension to more than two periods.
Implications for Consumer Education and Financial Counseling

Optimal saving and credit use depend on planned household size changes, as well as relative risk aversion and the elasticity of marginal utility with respect to consumption values. Telling a household how much to save or borrow is problematic from the perspective of expected utility theory and the results of this paper. However, if real income growth is not likely and children are planned in the future, advising substantial saving before the children come seems reasonable. Furthermore, advising heavy saving when children are at home does not seem reasonable.

References


Endnotes

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